

# QUANTUM EFFECTS DURING INFLATION

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The expansion of spacetime alters the energy-time uncertainty principle, allowing virtual particles to persist longer than in flat space. For a given expansion the effect is largest for massless particles which are not conformally invariant. For a given particle type the effect is largest for inflation. I exhibit these two principles in the context of massless fermions which are Yukawa-coupled to massless scalars. I also review the many related examples which have been studied recently.

## 1 Introduction

The invariant interval of a homogeneous, isotropic and spatially flat geometry can be written in the form,

$$ds^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x} . \quad (1)$$

Although the scale factor  $a(t)$  is not observable, its derivatives can be formed into the Hubble parameter and the deceleration parameter,

$$H(t) \equiv \frac{\dot{a}}{a} \quad , \quad q(t) \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} . \quad (2)$$

Inflation is defined as a period of accelerated expansion,<sup>1</sup> that is,  $H(t) > 0$  and  $q(t) < 0$ . Of fundamental importance to the theory of inflation is the conformal time interval,

$$\Delta\eta(t) \equiv \int_0^t \frac{dt'}{a(t')} . \quad (3)$$

During non-accelerated expansion  $\Delta\eta(t)$  grows without bound whereas it is bounded during inflation, even if the universe inflates forever. This simple fact is behind the inflationary resolution of the smoothness problem.<sup>1</sup> We will see that it also explains why certain particles experience large quantum effects during inflation.

Parker was the first to report quantitative results about how the expansion of spacetime affects virtual particles.<sup>2</sup> The physics is simple to understand in the context of the energy-time uncertainty principle and the fact that the energy of a particle with mass  $m$  and co-moving wave vector  $\vec{k}$  is,

$$E(t, \vec{k}) = \sqrt{m^2 + \|\vec{k}\|^2/a^2(t)} . \quad (4)$$

Three conclusions result:<sup>3,4</sup>

1. Growth of  $a(t)$  always increases the time a virtual particle of fixed  $m$  and  $\vec{k}$  can exist;
2. The persistence time for given  $a(t)$  and  $\vec{k}$  is longest for smallest  $m$ ; and
3. Virtual particles with  $m = 0$  can persist forever during inflation.

These principles govern what happens once a virtual particle emerges from the vacuum. Also important is the emergence rate, which depends upon the type of particle. Most massless particles are conformally invariant. This causes the rate at which they emerge from the vacuum to fall like  $1/a(t)$ .<sup>3,4</sup> So any particles that emerge can persist forever during inflation, but very few emerge. Hence conformally invariant particles engender no significant quantum effects during inflation.

Two familiar particles are both massless and also not conformally invariant: gravitons and massless, minimally coupled scalars. This means they can mediate large quantum effects during inflation. Starobinskii was the first to compute the contribution of inflationary gravitons to the cosmic microwave anisotropy.<sup>5</sup> This effect may be observable in the precision measurements of polarization planned for the next decade. Mukhanov and Chibisov were the first to suggest that fluctuations from inflationary scalars may have produced the tiny inhomogeneities needed to form the various cosmic structures of today under the influence of gravitational collapse.<sup>6</sup> The imprint of these fluctuations on the cosmic microwave anisotropy has been imaged with stunning precision by WMAP.<sup>7</sup>

A variety of related quantum effects have been studied from gravitons<sup>8,9</sup> and from scalars<sup>10,11,12,13,14,15</sup>. The most recent of these is the creation of massless fermions during inflation. These particles are conformally invariant so they experience no substantial particle creation by themselves. However, when Yukawa-coupled to a massless, minimally coupled scalar, there is copious production of fermions from the spontaneous appearance of a scalar and a fermion-anti-fermion pair.<sup>3</sup> In this paper I consider the same process for flat space, and for a conformally coupled scalar, in order to illustrate the crucial roles of inflation and conformal non-invariance.

## 2 Particle Creation in Yukawa Theory

Dirac fermions require gamma matrices  $\gamma^b$  which anti-commute as usual,  $\{\gamma^b, \gamma^c\} = -2\eta^{bc}I$ . Coupling fermions to gravity also requires the vierbein field,  $e_{\mu b}(x)$ , the contraction of two of which gives the metric,  $g_{\mu\nu} = e_{\mu b}e_{\nu c}\eta^{bc}$ . In a general vierbein background the action we wish to study is,

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\xi\phi^2 R\sqrt{-g} \\ & + \bar{\psi}e^\mu_b\gamma^b\left(\partial_\mu - \frac{1}{2}A_{\mu cd}J^{cd}\right)\psi\sqrt{-g} - f\phi\bar{\psi}\psi\sqrt{-g}. \end{aligned} \quad (5)$$

Here the spin connection and the Lorentz representation matrices are,

$$A_{\mu bc} \equiv e^\nu_b \left( e_{\nu c, \mu} - \Gamma^\rho_{\mu\nu} e_{\rho c} \right) \quad , \quad J^{bc} \equiv \frac{i}{4} [\gamma^b, \gamma^c] . \quad (6)$$

When  $\xi = \frac{1}{6}$  the scalar is conformally coupled; when  $\xi = 0$  it is minimally coupled.

For the special case of a homogeneous and isotropic metric (2) the associated vierbein can be taken to be,

$$e_{\mu b}(t, \vec{x}) \Big|_{\text{HI}} = -\delta_{\mu 0} \delta_{b0} + a(t) \delta_{\mu i} \delta_{bi} . \quad (7)$$

In this case the Lagrangian simplifies dramatically,

$$\begin{aligned} \mathcal{L} \Big|_{\text{HI}} = & \frac{a^3}{2} \left( \dot{\phi}^2 - \frac{1}{a^2} \vec{\nabla} \phi \cdot \vec{\nabla} \phi - \xi (12H^2 + 6\dot{H}) \phi^2 \right) \\ & + (a^{\frac{3}{2}} \bar{\psi}) \left( \gamma^0 \partial_0 + \frac{1}{a} \gamma^i \partial_i \right) (a^{\frac{3}{2}} \psi) - f a^3 \phi \bar{\psi} \psi . \end{aligned} \quad (8)$$

The conformal invariance of free fermions implies that (8) becomes even simpler when expressed in terms of conformally rescaled fields,

$$\Psi(t, \vec{x}) \equiv a^{\frac{3}{2}}(t) \psi(t, \vec{x}) \quad , \quad \bar{\Psi}(t, \vec{x}) \equiv a^{\frac{3}{2}}(t) \bar{\psi}(t, \vec{x}) . \quad (9)$$

Because (8) possesses spatial translation invariance the three free fields can be expanded in spatial plane waves,

$$\phi_I(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} \left\{ e^{i\vec{k} \cdot \vec{x}} A(t, k) \alpha(\vec{k}) + e^{-i\vec{k} \cdot \vec{x}} A^*(t, k) \alpha^\dagger(\vec{k}) \right\} , \quad (10)$$

$$\Psi_I(t, \vec{x}) = \int \frac{d^3 q}{(2\pi)^3} \sum_r \left\{ e^{i\vec{q} \cdot \vec{x}} B(t, \vec{q}, r) \beta(\vec{q}, r) + e^{-i\vec{q} \cdot \vec{x}} C(t, \vec{q}, r) \gamma^\dagger(\vec{q}, r) \right\} , \quad (11)$$

$$\bar{\Psi}_I(t, \vec{x}) = \int \frac{d^3 p}{(2\pi)^3} \sum_s \left\{ e^{i\vec{p} \cdot \vec{x}} \bar{C}(t, \vec{p}, s) \gamma(\vec{p}, s) + e^{-i\vec{p} \cdot \vec{x}} \bar{B}(t, \vec{p}, s) \beta^\dagger(\vec{p}, s) \right\} . \quad (12)$$

The various creation and annihilation operators are canonically normalized,

$$[\alpha(\vec{k}), \alpha^\dagger(\vec{k}')] = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') , \quad (13)$$

$$\left\{ \beta(\vec{q}, r), \beta^\dagger(\vec{p}, s) \right\} = \delta_{rs} (2\pi)^3 \delta^3(\vec{q} - \vec{p}) = \left\{ \gamma(\vec{q}, r), \gamma^\dagger(\vec{p}, s) \right\} . \quad (14)$$

The scalar wave function obeys a complicated equation,

$$\ddot{A}(t, k) + 3H \dot{A}(t, k) + \frac{k^2}{a^2} A(t, k) + \xi (12H^2 + 6\dot{H}) A(t, k) = 0 . \quad (15)$$

It can be solved for any  $a(t)$  in the conformally coupled case of  $\xi = \frac{1}{6}$ ,

$$\text{any } a(t) , \xi = \frac{1}{6} \quad \implies \quad A(t, k) = \frac{1}{\sqrt{2k} a(t)} e^{-ik\Delta\eta(t)} . \quad (16)$$

The general solution is also known for the minimally coupled case of  $\xi = 0$ ,<sup>16</sup> but it is sufficiently complicated that I shall specialize to the local de Sitter scale factor  $a(t) = e^{Ht}$ ,

$$a(t) = e^{Ht}, \xi = 0 \quad \implies \quad A(t, k) = \frac{1}{\sqrt{2k}} \left( \frac{1}{a(t)} + \frac{iH}{k} \right) e^{-ik\Delta\eta(t)}. \quad (17)$$

Of course the spinor wave functions are those of flat space expressed in terms of conformal time,

$$B(t, \vec{q}, r) = \frac{u(\vec{q}, r)}{\sqrt{2q}} e^{-iq\Delta\eta(t)}, \quad C(t, \vec{p}, s) = \frac{v(\vec{p}, s)}{\sqrt{2p}} e^{ip\Delta\eta(t)}. \quad (18)$$

The time evolution operator of the interaction picture is,

$$U \equiv T \left\{ \exp \left[ -if \int_{t_{\text{in}}}^{t_{\text{out}}} dt \int d^3x \phi_I(t, \vec{x}) \bar{\Psi}_I(t, \vec{x}) \Psi_I(t, \vec{x}) \right] \right\}. \quad (19)$$

We can think of the creation and annihilation operators as those relevant to the initial time  $t_{\text{in}}$ . The annihilators relevant to the final time  $t_{\text{out}}$  are,

$$\alpha^{\text{out}}(\vec{k}) = U^\dagger \alpha(\vec{k}) U, \quad \beta^{\text{out}}(\vec{q}, r) = U^\dagger \beta(\vec{q}, r) U, \quad \gamma^{\text{out}}(\vec{p}, s) = U^\dagger \gamma(\vec{p}, s) U. \quad (20)$$

Now consider the amplitude for the initial vacuum to produce a scalar and a fermion-anti-fermion pair,

$$\begin{aligned} & \left\langle \Omega \left| \sqrt{2k} \alpha(\vec{k}) \sqrt{2q} \beta(\vec{q}, r) \sqrt{2p} \gamma(\vec{p}, s) U \right| \Omega \right\rangle \\ &= -if (2\pi)^3 \delta^3(\vec{k} + \vec{q} + \vec{p}) \sqrt{8kqp} \int_{t_{\text{in}}}^{t_{\text{out}}} dt A^*(t, k) \bar{B}(t, \vec{q}, r) C(t, \vec{p}, s) + O(f^3), \end{aligned} \quad (21)$$

$$= -if (2\pi)^3 \delta^3(\vec{k} + \vec{q} + \vec{p}) \bar{u}(\vec{q}, r) v(\vec{p}, s) \int_{t_{\text{in}}}^{t_{\text{out}}} dt F(t) e^{i(k+q+p)\Delta\eta(t)} + O(f^3). \quad (22)$$

The factor  $F(t)$  in (22) is  $1/a(t)$  for  $\xi = \frac{1}{6}$  and  $(1/a(t) - iH/k)$  for  $\xi = 0$ .

It is simple to understand why the amplitude for this process is zero in flat space. For that case the scale factor is  $a(t) = 1$ , which implies the conformal time interval is  $\Delta\eta(t) = t$ . One also usually takes the initial and final times to  $\pm\infty$ , which results in a delta function that cannot be saturated for  $\vec{k} + \vec{q} + \vec{p} = 0$ ,

$$\text{Flat Space} \quad \implies \quad \int_{-\infty}^{\infty} dt e^{i(k+q+p)t} = 2\pi \delta(k+q+p). \quad (23)$$

Even with a finite time interval  $t_{\text{out}} - t_{\text{in}}$  the oscillations would still tend to cancel for intervals longer than  $1/(k+q+p)$ . This is the physics behind the energy-time uncertainty principle of flat space.

With  $\xi = \frac{1}{6}$  and arbitrary  $a(t)$  the integral becomes,

$$\text{Conformal Coupling} \quad \implies \quad \int_{t_{\text{in}}}^{t_{\text{out}}} \frac{dt}{a(t)} e^{i(k+q+p) \int_0^t \frac{dt'}{a'}}. \quad (24)$$

During inflation the phase factor approaches a constant, so there are no more oscillations. However, the integral is suppressed by the multiplicative factor of  $F(t) = 1/a(t)$ . This is why inflation gives only a slight enhancement of quantum effects for massless, conformally invariant particles.

With minimal coupling in a locally de Sitter background the integral is,

$$\text{Minimal Coupling} \implies \int_{t_{\text{in}}}^{t_{\text{out}}} dt \left( e^{-Ht} - \frac{iH}{k} \right) e^{i(k+q+p)(1-e^{-Ht})/H} . \quad (25)$$

At late times (i.e.,  $Ht \gg 1$ ) the integrand approaches a nonzero constant, so the integral grows linearly in  $t_{\text{out}}$ . Note also that a small mass  $m \ll H$  would not begin to produce oscillations until a time comparable to  $1/m$ .

### 3 Discussion

We have just studied the mechanism through which massless, minimally coupled scalars catalyze the production of massless fermions during inflation.<sup>3</sup> In a more complicated theory it is conceivable that this process might result in baryogenesis during inflation. A very similar calculation in massless, minimally coupled scalar QED gives strikingly different results. In that case the one loop vacuum polarization causes super-horizon photons to behave, in some ways, as if they possess nonzero mass.<sup>12,13,15</sup> Although there is no significant creation of photons during inflation, their 0-point energies are vastly enhanced. After the end of inflation some of this energy may end up seeding the cosmic magnetic fields we see in galaxies and galactic clusters.<sup>17,3</sup>

Massless, minimally coupled scalars with self-interactions can also do interesting things to gravity through the back-reaction from the stress-energy tensor. What happens for a  $\phi^4$  coupling depends upon the operator ordering scheme. If covariant normal-ordering is employed the three-loop back-reaction slows inflation by an amount that eventually becomes non-perturbatively strong.<sup>10</sup> The mechanism seems to be that inflation rips virtual scalars out of the vacuum, whereupon the attractive, long range interaction between these particles tends to pull them back together.

If time-ordering is used instead, the two-loop back-reaction increases the expansion rate.<sup>11</sup> The mechanism in this case seems to be that, as more and more virtual scalars are ripped out of the vacuum, the amplitude of the scalar field increases. This increases the potential energy from the  $\phi^4$  term, hence the expansion rate also increases. Starobinskiĭ and Yokoyama have shown that the effect is finally arrested by the classical force pushing the scalar towards the minimum of its potential.<sup>18</sup>

Although scalar effects are very interesting, they tend to be self limiting because secular contributions to the scalar mass also occur. This is guaranteed not to happen for gravitons, although the calculations are much more difficult. The two-loop back-reaction from graviton creation slows inflation by

an amount that eventually becomes non-perturbatively strong.<sup>8,9</sup> The mechanism seems to be the same as for covariantly normal-ordered scalars. It is conceivable that this process could quench  $\Lambda$ -driven inflation without the need for a scalar inflaton.

## Acknowledgments

It is a pleasure to acknowledge conversations and collaboration with L. R. Abramo, V. K. Onemli, T. Prokopec and N. C. Tsamis. This work was partially supported by DOE contract DE-FG02-97ER41029 and by the Institute for Fundamental Theory at the University of Florida.

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